

## Chapter 3 Groundwater movement

### 3.1 Darcy's Law

Natural porous media → permeability

→ Darcy Law (French hydraulic engineer, 1856)

$$Q \frac{L}{L}$$

$$Q \sim h_L \text{ (see Fig. 3.1.1 and sample 3.1.1)}$$

$$Q = -KA \frac{h_L}{L} = -KA \frac{dh}{dl}$$

$$V = \frac{Q}{A} = -K \frac{dh}{dl} \leftarrow \text{hydraulic gradient}$$

↑  
Darcy velocity  
Specific discharge

↑  
hydraulic  
conductivity

interstitial velocity

$$V_a = \frac{Q}{\alpha A} \quad d : \text{porosity} \cong 33\%$$

velocity

$$V_u = 3V \quad \text{actual velocity be quantification statistically}$$

Darcy's Law → laminar flow in porous media

$$N_R = P \frac{VD}{\mu}$$

P : fluid density    D : diameter  
V : velocity        μ : viscosity

$$= P \frac{VD_{10}}{\mu}$$

→  $1 < N_R < 10$  validity

3.2 Permeability : ability to transmit a fluid, property of the median dependent of fluid properties

Intrinsic permeability (n)

$$h = \frac{K_{\mu}}{\rho g}$$

$$= \frac{\mu v}{\rho g (dh / dl)} = \frac{(kg / ms)(m / s)}{(kg / m^3)(m / s^2)(m / m)} = m^2$$

*V.S Gewloinca Survey*  $n = (\mu m)^2 : \text{square ...croneter} = 10^{-12} m^2$

- Hydraulic Conductivity  
a unit hydraulic conductivity : transmit in unit time a unit volume of groundwater at the prevailing kinematic viscosity through a cross section of unit area, measured at right angles to the direction of flow, under a unit hydraulic gradient

$$K = -\frac{V}{dh / dl} = -\frac{m / day}{m / m} = m / day$$

- Transmissibility : rate of water prevailing kinematic viscosity transmitted through a unit width of aquifer under a unit hydraulic gradient

$$T = Kb = (m / day)(m) = m^2 / day$$

b=aquifer thickness

- Hydraulic Conductivity depends on a variety of physical factors including porosity, particle size and distribution, shape of particles, arrangement of particles and other factors

Fig 3.2.1  
Table 3.2.1

### 3.3 Determination of Hydraulic Conductivity

(by formulas, laboratory methods, tracer tests, auger hole test and pumping test)

$$K = cd^2 = f_s f_a d^2$$

$\leftarrow$  grain diameter  
 $\uparrow$  grain shape factor       $\swarrow$  porosity factor

Fig 3.2.2

- Laboratory methods

→ Constant head      Fig 3.3.1

→ Falling head

$$IC = \frac{VL}{Ath}$$

$$IC = \frac{r_c^2 L}{r_c^2 t} \text{ by } \frac{h_1}{h_2}$$

- Tracer Tests (field determination dye/salt Fig 3.3.2)

$$V_a = \frac{K h}{\alpha L}$$

$$V_a = \frac{L}{t}$$

$$\therefore K = \frac{\alpha L^2}{ht}$$

- Point Dilution Method (GW velocity, water table gradient Darcy's Law)

$$k = \frac{C}{864} \frac{dy}{dt}$$

- Anger Hole Tests Fig 3.3.1
- Pumping Test of Wells

### 3.4 Anisotropic Aquifers

(directional properties of Hydraulic conductivity)

$$K_x = \frac{K_1 Z_1 + K_2 Z_2}{(Z_1 + Z_2)}$$

Equivalent  
Horizontal  
Hydraulic  
Conductivity

$$q_z = K_1 \frac{dh_1}{Z_1}$$

$$K_Z = \frac{Z_1 + Z_2 + \dots + Z_n}{\frac{Z_1}{K_1} + \frac{Z_2}{K_2} + \dots + \frac{Z_n}{K_n}}$$

$$\frac{K_X}{K_Z} \geq 1$$

$$\frac{K_X}{K_Z} \cong 2 - 10 \text{ (for alluvium)}$$

$$\cong 100 \quad \text{clay layers are present}$$

### 3.5 Groundwater flow rates

$$K = 75 \text{ m/day}$$

$$V = Ki = 75 \text{ m/d} (0.01) = 0.75 \text{ m/day}$$

normal 2 m/year to 2 m/day

Fig 3.5.1

$$\text{pt } A \rightarrow B \quad V = K \frac{dh}{dl} = 10 \frac{27 - h_B}{27}$$

$$B \rightarrow C \quad V = K \frac{dh}{dl} = 0.2 \frac{h_B + 5 - 30}{5}$$

$$\therefore h_B = 26 - 8m$$

$$V = 0.07 \text{ m / day}$$

### 3.6 Groundwater flows Direction

→ Flow Net's (graphical, model studies)

$$Q = mg = \frac{Kmh}{n} \quad \text{Fig 3.10}$$

anisotropic media, flow lines and equipotential lines are not orthogonal except when the flow is parallel to one of the principal directions all horizontal dimensions are reduced by

$$K^1 = \sqrt{K_x K_z} \quad (\text{Fig 3.6.2}) \quad \text{transformed section}$$

→ Flow in relations to GW contours

- Flow net boundary setup dimension for anisotropic See Fig 3.6.4
- Estimate of local GW contours/flow directions Fig 3.6.5
- GW recharge/discharge
- Conductivity Fig 3.6.6

- Transmissivity (sample 3.6.2)

$$T = \frac{nQ}{mQ}$$

h : sh between closed contour...

$$\text{or } T = \frac{Q}{(L_1 + L_2)sh / sr}$$

- Flow across a water table

Fig. 3.6.9

$$\varepsilon = \tan^{-1} \left( \frac{K}{V_u} \tan \delta \right) - \delta$$

- Flow across a Hydraulic Conductivity Boundary

Fig 3.6.10

$$\frac{K_1}{K_2} = \frac{\tan \partial_1}{\tan \partial_2}$$

See Fig 3.6.12

- Regional Flow Patterns (Fig 3.6.13)  
accurate evaluation of GW. flows is contingent on a detailed knowledge of hydrogeologic conditions.  
**Local, intermediate and regional system.**

3.7 Dispersion : molecular diffusion **and** hydrodynamic mixing occurring with laminar flow through porous media.

Diffusion

$$\frac{\partial c}{\partial t} = D_L \frac{\partial^2 c}{\partial x^2} + D_T \frac{\partial^2 c}{\partial z^2} - V \frac{\partial c}{\partial x}$$

(homogeneous and isotropic) c : relative tracer concentration (...)

media for 2-D case  $D_L$  : Longitudinal

Fig 3.7.1

$D_T$  : Transverse dispersion coef.

- For hydrology - two fluids with different characteristics come in to contact
- pollutants into the ground
  - artificial recharge
  - salt intrusion

### 3.8 Groundwater Tracers

For evaluating directions and rate of GW flow under field conditions and to select reasonably satisfactory tracer.

### 3.9 General Flow Equations

$$V = -K \frac{\partial h}{\partial s}$$

$$q_{x,i} = -T_x W \left( \frac{\partial h}{\partial x} \right)_i = -K_x \left( \frac{\partial h}{\partial x} \right)_i$$

$$q_{x,o} = -T_x W \left( \frac{\partial h}{\partial x} \right)_o$$

Continuity Eq.

unit surface are  $S_s$  defined in section 2.8

coefficient

substituting

$$(q_{x,i} - q_{x,o}) + (q_{y,i} - q_{y,o}) = -S W^2 \frac{\partial h}{\partial t}$$

$$\frac{-T_x (\partial h / \partial x)_i - (\partial h / \partial x)_o}{w} - T_y \frac{(\partial h / \partial y)_o}{w} = -S \frac{\partial h}{\partial t}$$

$S_s$  specific storage vol. of water in unit of vol. of saturated aquifer release from storage for a unit decline in hydraulic head.

application to aquifers  
 gives analytical solution  
 with approximate B.C.

3.10 Unsaturated flow      downward vertical flow  
 (natural and artificial recharge)  
 upward vertical flow  
 (evaporation and transpiration)  
 movement of pollutants  
 horizontal flow in the capillary  
 zone.

Flow through unsaturated solids

Eq 3.10.7, 3.10.3

Table 3.10.1

### Unsaturated Hydraulic Conductivity

$$\frac{K_u}{K} = \left( \frac{S_s - S_0}{1 - S_0} \right)^3$$

Fig 3.26

$S_s$  : degree of saturation  
 $S_0$  : threshold saturation  
 Fig 3.10.4

$$\frac{K_u}{K} = \frac{a}{\frac{a}{b}(-h)^n + a}$$

Fig 3.10.5  
 a, b, h : constants : particle sizes .  
 h : pressure head

Vertical and Horizontal Flows → Fig 3.10.4

→ Downward water migration (rain fall, irrigation water).



- Above water table in the capillary zone flow rate decrease with the degree of saturation and hydraulic conductivity.
- The fraction of flow above the water table can be calculated from ...equivalent saturated thickness.

### 3.11 Kinematic Wave

Fig 3.11.1 individual soil water wave propagating downward through the soil under gravity drainage.

Rectangular portion : wetting front

- (1) Draining part (a)
- (2) Vertical plateau part (b)
- (3) Wetting front (c)

Eq. 3.11.1

Eq. 3.11.2

Eq. 3.11.3

### 3.12 Infiltration

Process of water penetrating into the soil

Fig. 3.12.1 moisture zone (saturated, transmission, wetting zone, wetting front)

Fig. 3.12.2 profile change as a function of time

Fig. 3.12.3 rainfall hyetograph with the infiltration rate and cumulative infiltration curve

Equation 3.12.8 Green-Ampt equation

(with concept in Fig 3.12.4) solve eq 3.12.10

Table 3.12.1 Green Ampt Infiltration Parameters for  
various soil classes

Sample 3.12.1